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# MONITORING OF STRUCTURAL SYSTEMS BY USING FREQUENCY DATA

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## SUMMARY

The present work evaluates the possibility of using dynamic data to assess structural integrity. It addresses the problem of understanding when it is sufficient to measure and use only natural frequencies, thus avoiding the need to measure modal shapes. The classic problem of detecting damage in beams, or beam assemblies, due to concentrated cracks, or damage spread over a structural member is dealt with. Damage is represented as a decrease in stiffness and linear behaviour before and after the event assumed to have caused damage is considered. Damage is restricted to a few unknown sections or elements, so that only the modification of few parameters of the system need to be determined. This study thus rejects assumptions unrelated to the physical aspects of the problem, in contrast to many papers on the subject. The amount of data to locate and quantify damage correctly is discussed; general considerations lead to the conclusion that a unique and reliable estimate of the damage can be obtained using only few additional frequency data with respect to the number of damaged zones. Continuous and discrete (finite element) models are examined. Finally the paper considers the applications to both analytical and experimental data of the procedure developed, which takes account of the peculiar characteristics of damage detection problem. 

KEY WORDS: monitoring; damage evaluation; structural identification; vibrating beams

## 1. INTRODUCTION

In the monitoring of structures, integrity is quantified by visual inspection and by carrying out a limited number of non-destructive tests, often based on dynamic techniques. The results obtained from dynamic tests at a low level of excitation are mainly natural frequencies, modal dampings and modal shape components. These are used to identify any decrease in the stiffness of the structural elements; information that is sometimes considered sufficient to verify the occurrence of damage.[1–15] For many civil and industrial engineering structures, damage to a structural element is first seen as a crack whose effects on mechanical behaviour are similar to those

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produced by a local reduction in stiffness. The monitoring of structures consists in comparing the results of tests carried out at various intervals, which allow to follow the evolution of damage. If the monitoring is carried out at regular intervals, it is reasonable to assume that at each testing a limited number of newly damaged elements or cracks will be detected; perhaps only one.

The problem of locating and quantifying damage has generally been viewed as a reconstruction problem, where the distribution of the stiffness parameters for the whole structure is unknown and the solution requires quantity of data that is impossible to obtain in practice.[16] Recent papers on vibrating beams have pointed out that the solution requires in fact the evaluation of two quantities: the position and the degree of damage. The problem is very similar when a discrete model is considered, although this has never clearly been recognized. The peculiarity of damage detection as compared with a classical reconstruction problem, is precisely the fact that only a few parameters need to be determined, since the damaged elements are very few, in number, albeit unknown. The desired solution is such that the stiffness is known throughout to be equal to the undamaged value, except in the few damaged zones.

The present work evaluates the possibility, using dynamic data, to monitor structures. It is important to understand under which conditions it may be sufficient to measure and use only natural frequencies, avoiding the measurement of modal shapes. A full understanding of this aspect will facilitate monitoring since the measurement of frequencies alone is certainly attractive. The paper starts by describing a methodology already partially developed by the authors, [14,15,17] which suggests a strategy for detecting damage in an arbitrary, but small, number of elements. Discrete and continuous systems are dealt with and their differences are evidenced. For the discrete systems the identification procedure is carried out by the computer code IDEFEM,[18] which includes a finite element general purpose code as routine and performs all the steps needed to obtain the best estimate of the parameters. Continuous systems are either modelled as discrete or studied in closed form depending on their complexity. It is shown that generally the problem can be dealt with in two stages: in the first the damaged zones are located; in the second the degree of the damage is evaluated. Various objective functions are considered and account is taken of the lack of information concerning the undamaged state of the structure and the number of damaged zones. The procedures developed are then applied to evaluate damage in a simple supported beam, a three-span continuous beam and a ten storey shear-type frame; in these cases noise-free situations are considered. Lastly, two experimental cases are considered, in which errors are naturally present.

## 2. BASIC ASPECTS OF DAMAGE EVALUATION

From a theoretical point of view it is convenient to distinguish between continuous and discrete structures. Although all structures are in fact continuous, dynamic analysis considers structures in which concentrated masses are dominant as being discrete. Thus frames are seen as discrete structures while bridge decks and pipelines are considered continuous. For the continuous structures it is conceptually correct to pose the problem of localizing of a crack, because its position affects the entire dynamics. For a discrete structure the problem is different; for example it is not possible to locate a crack in a massless column of a frame, because the dynamics of the structure are influenced by the stiffness of the whole column with which infinite positions of the crack can be associated. In the context of damage identification, structures are considered as discrete if the damage cannot affect a portion smaller than the element.

 

From an algorithmic point of view it may sometimes be convenient or necessary to discretize continuous structures and to use a single approach for both continuous and discrete structures. It must, however be kept in mind that by decreasing the size of the discretization, some general statements obtained for discrete systems may not remain valid.

### 2.1. Discrete systems

A discrete system is often modelled with finite elements. Damage to the system will be represented by a decrease in the stiffness of the single finite elements. It is assumed here that the stiffness matrix of the whole element decreases uniformly, and its variation is expressed as:

$$\Delta \mathbf{K}_i = x_i \mathbf{K}_i^{\mathrm{u}}, \quad i = 1,2, \ldots, n \tag{1}$$

where $\mathbf{K}_i^{\mathrm{u}}$ is the undamaged stiffness matrix of the $i$th element, $x_i$ is a real number ranging in 0–1 and $n$ is the total number of elements.

Assuming that $m$ natural frequencies grouped in the vector $\lambda$ are known, the problem of damage evaluation is represented by an equation of the kind

$$\lambda - \mathbf{h}(x) = 0, \quad \lambda \in R^m, \quad \mathbf{x} \in R^{n+}, \quad \| \mathbf{x} \| \leqslant 1 \tag{2}$$

where $\mathbf{x}$ is the vector collecting the stiffness variation $x_i$ of all $n$ finite elements and $\| \cdot \|$ is the absolute norm. With this norm the condition $x_i \leqslant 1$, $i = 1,2, \ldots, n$, can simply be indicated by $\| \mathbf{x} \| \leqslant 1$. Equation (2) requires at least $m = n$ frequencies to have a finite (or a countable infinity) number of isolated solutions and $m = n + 1$ frequencies to have only one solution. The problem cannot generally be solved in practice because only a few frequencies can be measured in experimental situations and $m$ is much lower than $n$.

When, as usual, the damage is localized only on a limited number $r$ of elements, that is only a limited number of components of $\mathbf{x}$ are different from zero, though without knowing which they are, the problem can be stated as:

$$\boldsymbol{\lambda} - \mathbf{h}_\alpha(\mathbf{x}, \alpha) = 0, \quad \lambda \in R^m, \quad \mathbf{x} \in R^{r+}, \quad \| \mathbf{x} \| \leqslant 1, \quad \boldsymbol{\alpha} \in I^r \tag{3}$$

where $\boldsymbol{\alpha}$ is a vector of integers containing the number of damaged finite elements and the function $\mathbf{h}_\alpha(\mathbf{x}, \boldsymbol{\alpha})$ is obtained from $\mathbf{h}(\mathbf{x})$ where the components of $\mathbf{x}$ of undamaged elements, whose number is not contained in $\boldsymbol{\alpha}$, are set to zero. It is shown in Reference 14, that a unique solution exists for $m \geqslant r + 1$; a heuristic argumentation is reported below which shows the fact that can easily be extended to the continuous situation.

Although not strictly necessary, the assumption that the function $\mathbf{h}_\alpha(\mathbf{x}, \boldsymbol{\alpha})$ is weakly non-linear in $\mathbf{x}$ is made and the problem (3) is then replaced by

$$\Delta\boldsymbol{\lambda} - \mathbf{H}_\alpha \mathbf{x} = 0, \quad \boldsymbol{\lambda} \in R^m, \quad \mathbf{x} \in R^{r+}, \quad \| \mathbf{x} \| \leqslant 1, \boldsymbol{\alpha} \in I^r \tag{4}$$

where $\mathbf{H}_\alpha$ is the $m \times r$ Jacobian matrix of $\mathbf{h}(\mathbf{x}, \boldsymbol{\alpha})$ depending on the integer vector $\boldsymbol{\alpha}$ and $\Delta\boldsymbol{\lambda}$ is the variation of $\boldsymbol{\lambda}$ with respect to the undamaged state. The linearity assumption furnishes a simple procedure for finding the damaged elements directly prior to ascertaining the degree of damage. For $m = r$ linear system (4) has a solution $\mathbf{x}$ for each of the $\binom{n}{r}$ combinations of $\boldsymbol{\alpha}$ values. For $m > r$ the system admits solutions on condition that:

$$\operatorname{rank}\,(\Delta\lambda \,|\, \mathbf{H}_\alpha) = \operatorname{rank}\, \mathbf{H}_\alpha \tag{5}$$

 

which means that when $m = r + 1$ and $(\Delta\boldsymbol{\lambda} \,|\, \mathbf{H}_\alpha)$ is an $m \times m$ square matrix, the solvability conditions require that the following equation must be verified:

$$\det(\Delta\boldsymbol{\lambda} \,|\, \mathbf{H}_\alpha) = 0. \tag{6}$$

Equation (6) has $\boldsymbol{\alpha}$ as unknowns; for all possible sets of $r$ damaged elements $\mathbf{H}_\alpha$ is evaluated and the set for which the determinant $(\Delta\boldsymbol{\lambda} \,|\, \mathbf{H}_\alpha)$ vanishes supplies the location of the damaged elements, without the need to know the degree of damage. When $\Delta\boldsymbol{\lambda}$ is arbitrary, because of the discrete nature of the problem, no vector $\boldsymbol{\alpha}$ generically exists which satisfies it. The solution exists only if $\Delta\boldsymbol{\lambda}$ corresponds to a real situation, i.e. if there are $\boldsymbol{\beta} \in I^r$ and $\mathbf{y} \in R^{r+}$ such that $\Delta\boldsymbol{\lambda} = \mathbf{H}_\beta \mathbf{y}$. Note that equation (6) gives spurious solutions for $\boldsymbol{\alpha}$ values for which $\det \mathbf{H}_\alpha = 0$. This occurs, for example, when $\boldsymbol{\alpha}$ contains some coincident numbers, because fewer than $r$ distinct damaged elements are considered; this inconvenience can however be avoided.

In conclusion, it is possible to locate a few cracks with a number of frequencies depending on $r \ll n$ and because $m$ is often greater than $r$, the problem of damage can easily be solved.

### 2.2. *Continuous systems*

For continuous systems, the above considerations regarding the amount of data necessary to locate and evaluate damage are not wholly valid, also for $r = 1$. It has been shown for example[15] that to be sure of a unique solution in locating one crack in a simply supported beam, three frequencies are necessary, rather than two, as might be expected if the beam were a discrete system.

The problem of locating $r$ cracks in a continuous system may be posed as:

$$\boldsymbol{\lambda} - \mathbf{h}(\mathbf{x}, \mathbf{s}) = \mathbf{0}, \quad \boldsymbol{\lambda} \in R^m, \quad \mathbf{x} \in R^{r+}, \quad \mathbf{s} \in R^r \tag{7}$$

where $\mathbf{s}$ is the vector containing the crack positions and $\mathbf{x}$ is the vector of the stiffness parameters. The problem is formally equivalent to that reported in equation (3), with the vector of continuous variables $\mathbf{s}$ replacing $\boldsymbol{\alpha}$. The linearized problem becomes:

$$\Delta\boldsymbol{\lambda} - \mathbf{H}(\mathbf{s})\mathbf{x} = 0, \quad \boldsymbol{\lambda} \in R^m, \quad \mathbf{x} \in R^{r+}, \mathbf{s} \in R^r \tag{8}$$

where $\mathbf{H}$ is a matrix whose elements can be obtained on the basis of the known eigenfunctions.

The linear system (8) for $m \leqslant r$ admits an uncountable infinity of solutions (for each $\mathbf{s}$ at least one $\mathbf{x}$ can be obtained). For $m > r$ there is the solvability condition (5) that for $m = r + 1$ implies:

$$\det(\Delta\lambda \,|\, \mathbf{H}(\mathbf{s})) = 0 \tag{9}$$

Because $\mathbf{s}$ is an $r$-dimensional continuous variable, equation (9) is a non-linear equation in the classical sense and as such it admits an $r - 1$ dimensional submanifold of solutions in $R^{r+}$. To have isolated solutions, $r - 1$ frequencies must also be known in addition. To have a unique solution one more frequency is generically sufficient, so that for a unique evaluation of $r$ crack locations and degree of damage, at least $2r + 1$ frequencies are necessary; only occasionally $2r$ may be sufficient.

 

## 3. IDENTIFICATION PROCEDURE

In the applications, the frequencies and, consistently, the eigenvalues, are available with experimental errors and the analytical model, in particular, is not sufficiently accurate. To reduce the bias of these errors, through the use of averaging operations, a larger number of eigenvalues than that strictly necessary to solve the damage detection problem must be used. Moreover the number of cracks or damaged elements and the stiffness distribution in the undamaged state are not always known *a priori*. The theoretical arguments developed above must then be adapted to the real world using system identification methods, where the optimal estimate of parameters is obtained by the minimization of an objective function. Here reference is made mainly to discrete systems, both for the sake of simplicity and because a continuous system can be studied as though it were discrete.

For $r$ damaged elements the objective function is assumed in the form:[14]

$$l(\mathbf{x}, \alpha) = \| \boldsymbol{\lambda} - h_{\alpha}(\mathbf{x}, \boldsymbol{\alpha}) \|^2_{\Sigma_n}, \quad \boldsymbol{\lambda} \in R^m, \quad \mathbf{x} \in R^{r+}, \quad \boldsymbol{\alpha} \in I^r \tag{10}$$

where $\Sigma_n$ is a normalizing matrix that defines the norm $\|\mathbf{a}\|^2 = \mathbf{a}^{\mathrm{T}} \Sigma_n^{-1} \mathbf{a}$ and assumes the meaning of error covariance matrix when the experimental and analytical errors have a known statistical distribution. Unluckily this is not a frequent case and usually $\Sigma_n$ is assumed to be diagonal with coefficients estimated with engineering judgement.

The objective function (10) is minimized in two steps; in the first step the variable $\mathbf{x}$ is condensed, the new objective function is obtained:

$$\tilde{l}(\boldsymbol{\alpha}) = \min_{x \in R^{r+}} l(\mathbf{x}, \boldsymbol{\alpha}), \quad \boldsymbol{\alpha} \in I^r \tag{11}$$

the minimum of which locates the damage at $\hat{\boldsymbol{\alpha}}$. In the second step the degree of damage $\mathbf{x}$, associated with $\hat{\boldsymbol{\alpha}}$ is recovered from the first step.

Objective function (11) is well defined and can efficiently be computed because $\mathbf{h}(\mathbf{x})$ is weakly non-linear and $l(\mathbf{x}, \boldsymbol{\alpha})$ is strictly convex in $\mathbf{x}$ over a large domain of the space parameters around the undamaged state. For a continuous system an equivalent objective function can be defined by replacing $\boldsymbol{\alpha}$ with $\mathbf{s}$.

### 3.1. *Alternative formulation of the objective function*

With some additional expense an alternative objective function can be constructed by inverting the function $\lambda = \mathbf{h}(\mathbf{x}, \boldsymbol{\alpha})$ with respect to $\mathbf{x}$, for an assigned set of $r$ eigenvalues. By indicating with

$$\mathbf{x}_i(\boldsymbol{\alpha}) = \mathbf{g}(\boldsymbol{\lambda}_i, \boldsymbol{\alpha})$$

the parameter vector $\mathbf{x}$ associated to the $i$th set $\boldsymbol{\lambda}_i$ of $r$ eigenvalues and the assigned location defined by $\boldsymbol{\alpha}$, the following objective function can be used:

$$\hat{l}(\boldsymbol{\alpha}) = \sum_{i,j} \| \mathbf{x}_i - \mathbf{x}_j \|^2_{\Sigma_x} = \sum_{i,j} \| \mathbf{g}(\boldsymbol{\lambda}_i, \boldsymbol{\alpha}) - \mathbf{g}(\boldsymbol{\lambda}_j, \boldsymbol{\alpha}) \|^2_{\Sigma_x} \tag{12}$$

where $\Sigma_x$ is a diagonal matrix with elements given by the square of the expected values. Relation (12) is motivated by the fact that different sets of mesurements should give the same value for the parameters $\mathbf{x}$ at the correct location $\boldsymbol{\alpha}$. An application of this objective function can be found in Reference 15 for $r = 1$ and a simply supported beam.

 

When the linearity assumption of $\mathbf{h}(\mathbf{x})$ versus $\mathbf{x}$ is accepted the objective function can be derived by the condition on the rank of matrix $\mathbf{H}_\alpha$. It can be written as

$$\underline{l}(\boldsymbol{\alpha}) = \sum_i [\det(\Delta\boldsymbol{\lambda}_i \,|\, \mathbf{H}_\alpha^i)]^2 \tag{13}$$

where $\boldsymbol{\lambda}_i$ is the $i$th set of $(n+1)$ eigenvalues and $\mathbf{H}_\alpha^i$ is the $(n+1)\times r$ submatrix of $\mathbf{H}_\alpha$ with rows corresponding to $\lambda_i$.

### *3.2. Number of damaged elements and undamaged state unknown*

When the number of damaged finite elements or cracks is unknown, the problem of identifying the damage becomes more difficult. If the covariance matrix $\Sigma_n$ of the error is not known with some accuracy, only an empirical criterion to stop can be established, consisting in interrupting the analysis when, on increasing $r$, the minimum values of $l(\hat{\mathbf{x}}, \boldsymbol{\alpha})$ do not decrease substantially and, above all, for an assumed number of parameters some of them are close to zero. Because in most cases the number of damaged elements is expected to be very small the procedure is practicable.

Generally the undamaged stiffness distribution is not known with sufficient accuracy. This does not invalidate the strategy proposed, concerning the identification of a few damaged elements at a time, even though to know the undamaged state it is necessary to solve a reconstruction problem, which is claimed to avoid. There is, however, a simple but often highly effective means to adjust things. The undamaged state is identified as correctly as possible from the given experimental data, which are generally not sufficient for a reconstruction, and a priori assumption is made on the stiffness distribution. A comparison is then drawn not directly between the experimental and analytical eigenvalues in the damaged state, instead the following objective function is assumed:

$$l(\mathbf{x}, \mathbf{s}) = \| \Delta\boldsymbol{\lambda} - \Delta\mathbf{h}(\mathbf{x}, \mathbf{s}) \|^2_{\Sigma_n} \tag{14}$$

where $\Delta\boldsymbol{\lambda}$ and $\Delta\mathbf{h}$ are experimental and analytical frequency variations, respectively, between the damaged and undamaged states. Naturally this procedure can eliminate only a part of the modelling error associated with a wrong assumption of the undamaged state, but it is often sufficient.

## 4. APPLICATIONS

### *4.1. Simply supported beam*

The supported beam has already been studied by the authors both as a discrete,[14] and as a continuous system.[15] For the sake of clarity, some of the results obtained are resumed here, though reference is mainly made to the developments of the present paper. Results are obtained by assuming a linearization of the eigenvalues–parameters law; this is sufficiently accurate to show the qualitative aspects of the problem. In the following, location of a crack is considered only as a separate step in the complete process, where the degree of damage is also of interest; this

 

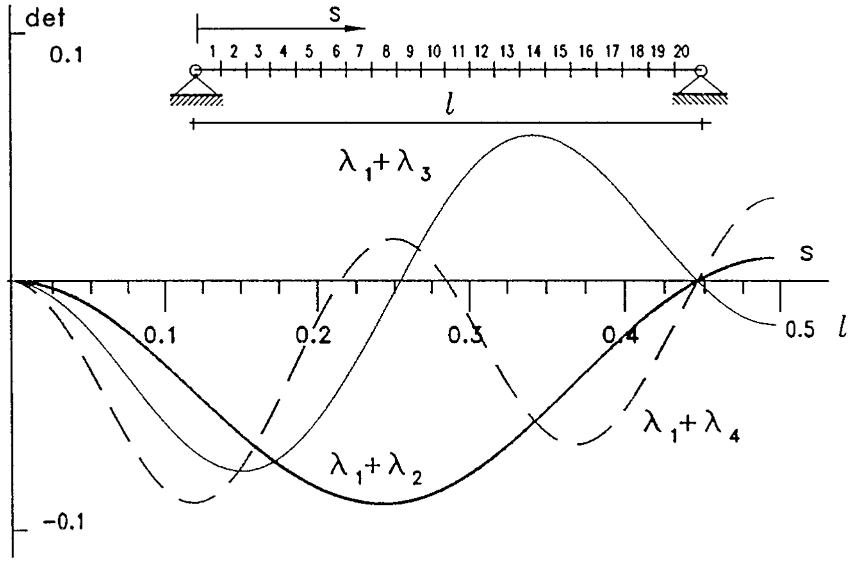


Figure 1. Multiplicity in the damage location with two frequencies

is also valid for the next application, a continuous beam. Figure 1 shows the trend of the function $\det(\Delta\boldsymbol{\lambda}\,|\,\mathbf{H}(s))$, for a crack at $s = 0{\cdot}45$ and three different frequency couples. It is clear from the figure that more than one solution may occur, apart from the case where the couple $\lambda_1$ and $\lambda_2$ are used; in particular if the first and third frequencies are assumed there will be two solutions; if the first and fourth frequencies are assumed there will be three solutions and, not shown in the figure, if the first and the tenth frequencies are assumed there will be eight solutions. It is clear from the figure that when any set of three frequencies are assumed, no matter which, a unique solution is obtained in the generic sense.

Figure 2 refers to a situation with two cracks, located at ($s_1 = 0{\cdot}45$, $s_2 = 0{\cdot}2$), similar to the case analysed in Reference 14 where the beam was assumed as the prototype of a discrete structure. In Figure 2(a) the contour lines of the function $|\det(\Delta\boldsymbol{\lambda}|\mathbf{H}(s_1,\ s_2))|$ are reported by assuming $\Delta\boldsymbol{\lambda}^{\mathrm{T}} = (\Delta\lambda_1\ \Delta\lambda_2\ \Delta\lambda_3)$, in Figure 2(b) the same function is reported for $\Delta\boldsymbol{\lambda}^{\mathrm{T}} = (\Delta\lambda_1\ \Delta\lambda_2\ \Delta\lambda_4)$. Bold lines give curves of zeros. Lines $s_1 = s_2$ and $s_1 = 0, s_2 = 0$, where the two cracks degenerate in one only, must be ignored, because for them the matrix $\mathbf{H}(s_1, s_2)$ becomes singular and the solvability condition (5) cannot be satisfied for arbitrary $\Delta\boldsymbol{\lambda}$; condition (5) can of course give the correct location, when $\Delta\boldsymbol{\lambda}$ derives from one crack only. The two families of curves of zeros meet only at the right point (Figure 2(a)). In this case four frequencies ($m = 2r$) are sufficient for a correct location.

### 4.2. *Three-span continuous beam*

A more complex example is the three-span continuous beam discretized in 72 elements, reported in Figure 3(a) where the damage again affects only one element. The objective function $\tilde{I}(s)$ is evaluated for each $s_i$ position of damage, $i = 1{,}72$, with the IDEFEM code by using different

 

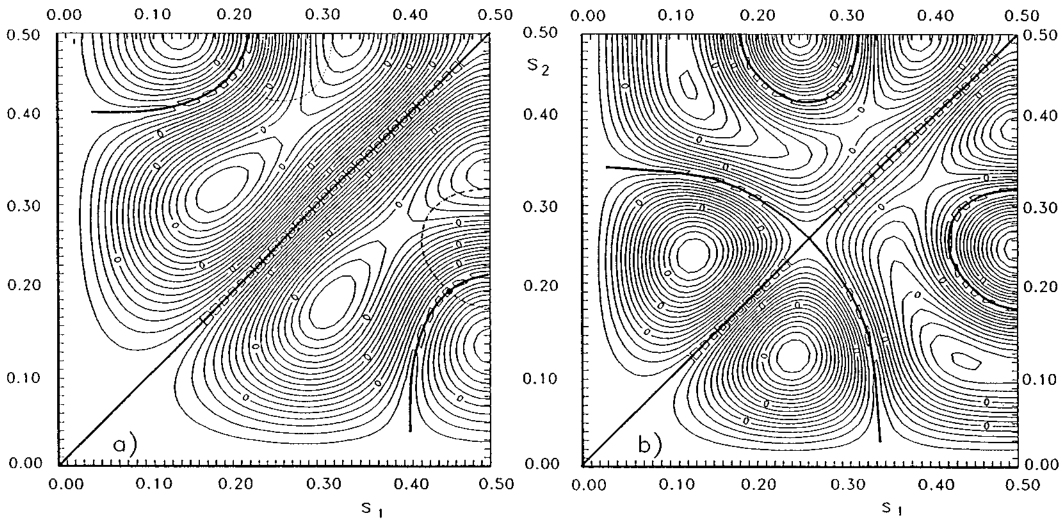


Figure 2. Contour lines of $|\det(\lambda\,|\,\mathbf{H}(\mathbf{s}))|$ for two cracked sections

sets of the first eight frequencies and for two different damage scenarios. In the first scenario the damaged element is the 16th, in the second it is the 30th; in both cases the degree of damage is $x = 0{\cdot}5$. The structure, though continuous, is modelled as discrete so that the aspects of both continuous and discrete cases are present.

Figure 3(c) reports, in logarithmic scale, the objective function $\tilde{l}(s)$ for the first damage scenario. As can be seen two frequencies (the first and the second are considered in the figure) are sufficient to estimate exactly the damage, as shown for discrete systems. However, not only the objective function had an absolute minimum which is not exactly zero on account of rounding errors, but also reveals some very evident local minima which betray the continuous nature of the problem. Indeed, an analysis of the three-span beam under consideration, as a continuous system, using as data the first two frequencies corresponding to a crack centred in the 16th element, reveals the existence of numerous other isolated solutions for crack locations, together with that centred around the 16th element. These solutions do not give zeroes for the objective function in the discrete case only because of discretization errors. When more frequencies than those strictly necessary are used (Figure 3(c)) all the evident local minima, less one, disappear, consistently with the fact that the continuous case now has only one solution.

When a less refined model is used (Figure 3(b)) that cannot reproduce accurately the actual damaged zone, modelling errors are introduced, and consequently more frequencies are required in order to reach a unique solution. This is illustrated in Figure 3(d) which shows the objective function $\tilde{l}(s)$ for a model with 30 parameters, rather than 72, obtained by the original model where a parameter is considered for more than one element. This suggests the possibility of an analysis in two phases with a refined discretization in the zones where local minima are found.

Figures 3(e)–3(f) show the results for the second damage scenario. The general aspects are as previously, but $\tilde{l}(s)$ has a more regular trend with fewer local minima, suggesting that to a certain extent the solvability depends also on the specific case.

 

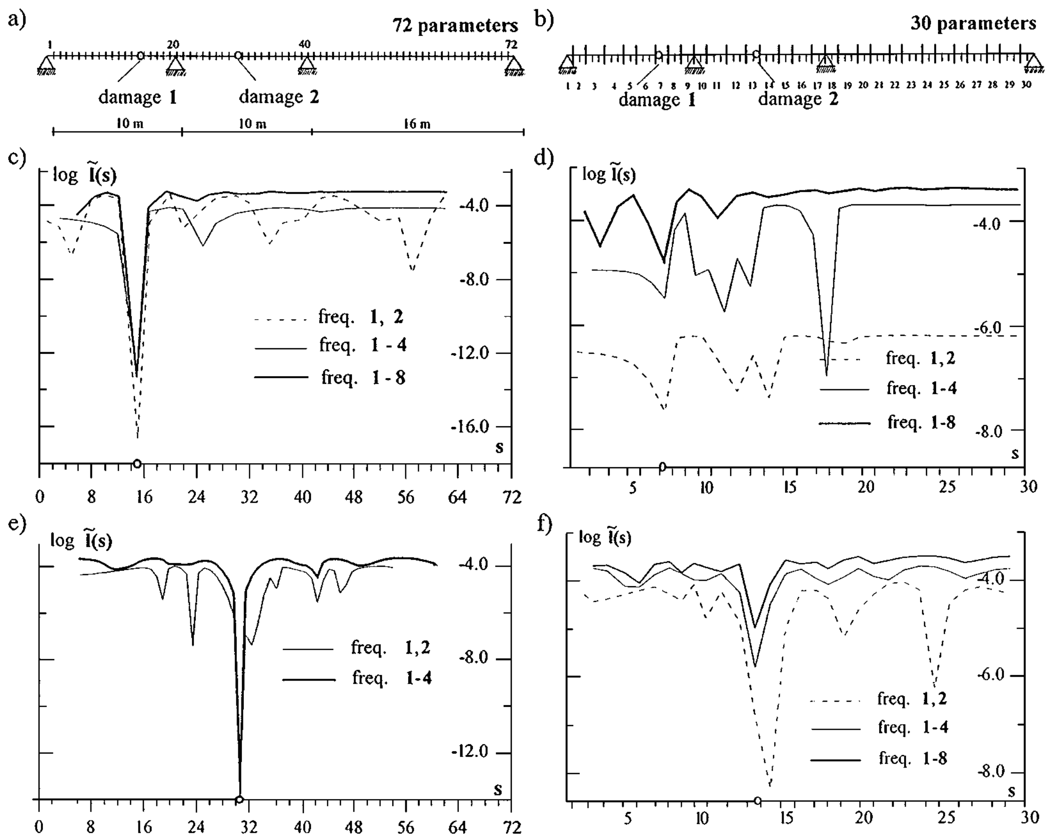


Figure 3. Geometry and objective function $\tilde{I}(s)$ for the continuous beam

*4.3. Shear-type building model*

The identification of damage in a shear-type model is very similar to that in a clamped beam, already analysed in Reference 14. There is however one important difference: the shear-type model is typically a discrete structure, since the column masses are negligible in comparison with the floor masses, while the clamped beam is a continuous system which can be discretized at different levels of refinement. The same model as that assumed in Reference 13 is considered; it is shown in Figure 4, with the appropriate mass and stiffness characteristics. Because this is a discrete structure, there is no point in looking for a crack location. The problem instead is to evaluate the decrease in the stiffness of the columns of a certain number of floors. Three damage scenarios are created (Figure 4), two of them already considered by Topole and Stubbs,[13] are characterized by different locations and entities of damage. The identification is carried out by using the objective function $\tilde{I}(\boldsymbol{\alpha})$ which is evaluated for each $\alpha$ with the IDEFEM code.

For the first scenario, with two damaged elements and the first three frequencies known, it is possible to represent the objective function $\tilde{I}(\boldsymbol{\alpha})$, which assumes only discrete values. This

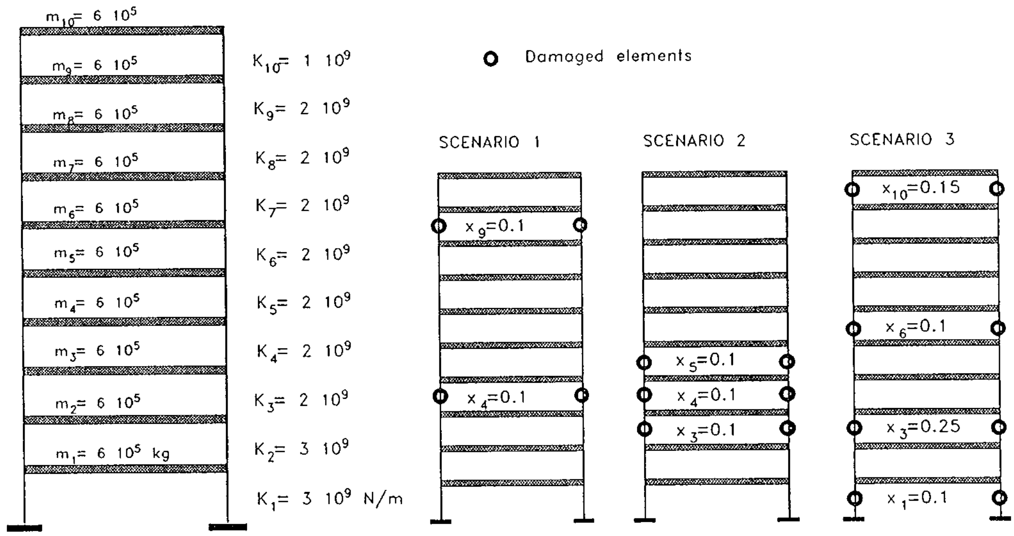


Figure 4. Geometry of the shear-type frame and the damage scenario

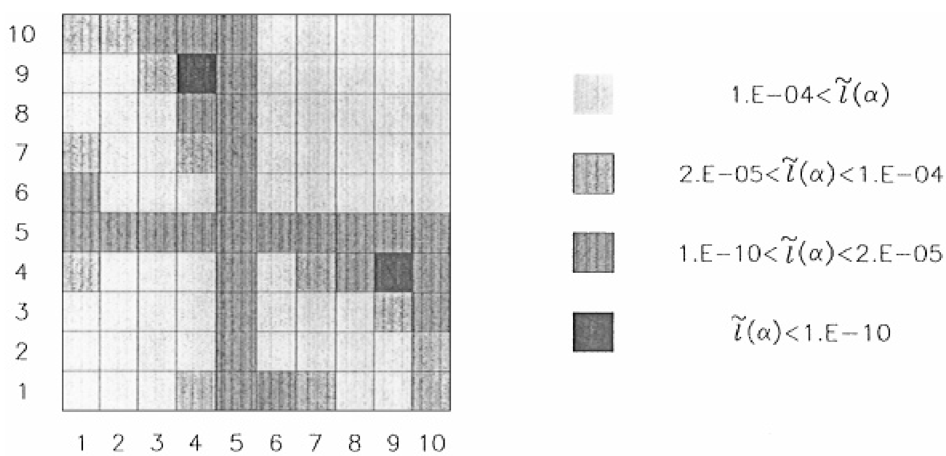


Figure 5. Objective function for the shear-type frame with two damaged elements

function, shown in Figure 5, is symmetric in the domain spanned by ($i$ = 1–10) and ($j$ = 1–10), because the order of the damaged element is not essential and then $\tilde{l}(i, j) = \tilde{l}(j, i)$. In the half-domain ($i$ = 1–10; $j$ = $i$–10), $\tilde{l}(i, j)$ has only one well-defined minimum at the exact location of damage ($i = 4$, $j = 9$). The evaluation of all possible couples among the 55 candidates makes it possible to check also if the case with one damaged element is possible ($i = j$). The objective function $\tilde{l}(i, i)$ attains a relative minimum on the 5th element, but its value is much higher than that attained by assuming two damaged elements.

For the second scenario, with three damaged elements, four frequencies are considered, while for the third scenario with four damaged elements five frequencies are considered as known; in

 

Table I. Objective function for various $r$ values for the second damage scenario

| $r$ | Location | Degree | $\tilde{l}(\alpha)$ |
|---|---|---|---|
| 1 | 4 | 0·7788 | $2{\cdot}7\times10^{-5}$ |
| 2 | 4, 5 | 0·6667, 0·7791 | $1{\cdot}3\times10^{-5}$ |
| 3 | 3, 4, 5, | 0·8999, 0·9002, 0·8999 | $5{\cdot}0\times10^{-10}$ |

both cases the strategy of gradually increasing the number $r$ of damaged elements to assume is applied. In the second scenario, where the damage is localized in contiguous elements, for $r=1$ and $r=2$ the objective function is minimum for the element 4, the centre of the damaged zone. The high value of the objective function, which should be close to zero at the point where damage is correctly identified, suggests increasing the number of possible damaged elements. For $r=3$, where $\binom{10}{3}=120$ combinations of $\boldsymbol{\alpha}$ are considered, a unique value, the correct one, $\boldsymbol{\alpha}^{\mathrm{T}}=(3\,4\,5)$ exists where the objective function practically vanishes. Table I refers to the number of elements identified as damaged, the degree of damage and the value for the objective function by increasing $r$.

For the third scenario, where the damage is distributed all over the frame, results for $r<4$ are not shown because they are not physically relevant. However the strategy of increasing $r$ furnishes the correct solutions. There are now 210 combinations of $\boldsymbol{\alpha}$ for $r=4$, but because of the simplicity of the structure under consideration this does not represent a problem.

### *4.4. Experimental cases*

Two experimental situations are examined, the first from the literature and the second obtained by Vestroni *et al.*[15] Among the cases of the literature which use experimental data, that presented by Stubbs and Osegueda,[5] already discussed in References 14 and 15, is very useful to illustrate the effectiveness of the procedure and, at the same time, the high sensitivity to errors of the problem. The authors examined a clamped beam with damage caused by reducing the section height over a portion equal to 1/10 of the span (Figure 6). The analytical model adopted is discrete (FE model). Various damage locations and intensities were studied in Reference 5; here the case study 8 is investigated, in which a stiffness reduction of 30% in the zone corresponding to the 5th element of the FE model is inflicted. Table II reports experimental frequencies for damaged and undamaged conditions together with the undamaged values identified. These values are obtained by considering a model with uniform constant properties and all the measured frequencies; the differences between experimental and identified values (5th column) indicate that modelling and experimental errors are of the same magnitude as the variations due to damage. Consequently the objective function (14) is preferred.

Figure 6 shows the objective function $\tilde{l}(s)$ evaluated using two, three and six frequencies. The position of the damaged zone is always correct, while the stiffness reduction in the 5th element is found to be between 17 and 27 per cent, instead of 30 per cent as it should be; even the use of six frequencies, a much higher number than that strictly necessary, does not furnish a more approximate value for damage intensity. Since two frequencies are sufficient to identify a damaged element, an analysis is developed to select a satisfactory pair of experimental data. The

 

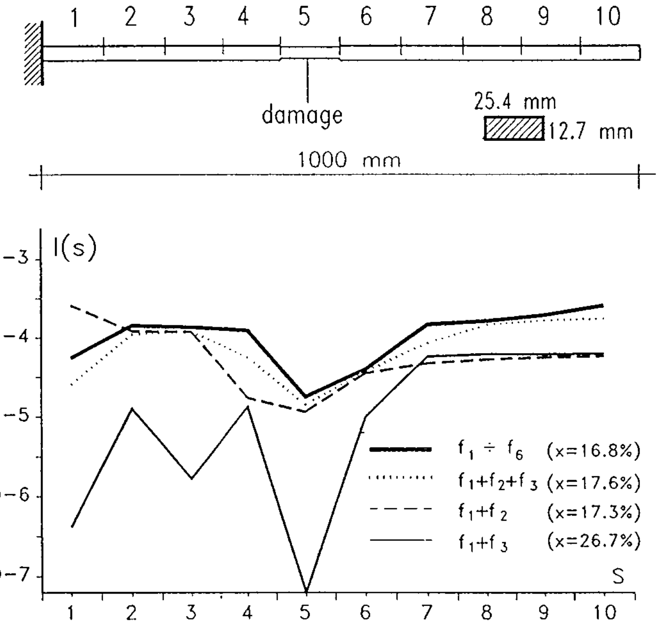


Figure 6. Objective function $\tilde{I}(s)$ for the clamped beam and experimental data

Table II. Experimental ($f_e$) and identified ($f_a$) frequencies of the clamped beam (Hz) for damaged and undamaged situations

| Mode | $f_e^U$ | $f_e^D$ | $\dfrac{f_e^U - f_e^D}{f_e^U}$ | $f_a^U$ | $\dfrac{f_a^U - f_a^U}{f_e^U}$ | $\dfrac{f_a^U - f_a^D}{f_a^U}$ |
|---|---|---|---|---|---|---|
| 1 | 9·375 | 9·276 | 1·13 | 9·513 | − 1·75 | 1·34 |
| 2 | 59·06 | 58·11 | 1·62 | 59·45 | − 0·87 | 3·43 |
| 3 | 165·9 | 165·1 | 0·48 | 166·0 | − 0·14 | 0·47 |
| 4 | 328·9 | 325·6 | 1·19 | 324·5 | 1·34 | 2·32 |
| 5 | 531·8 | 528·2 | 0·72 | 535·0 | − 0·39 | 1·24 |
| 6 | 809·1 | 804.0 | 0·63 | 796·9 | 1·94 | 0·86 |

use of different combinations of available frequencies always indicated the damage as being located in the 5th element; by assuming the damage to be located in this element, the variations of frequencies, furnished by the finite element model for an intensity of damage equal to the value that best fits all the experimental data are determined and reported in the last column of Table II. When the third and the sixth columns are compared it can be noticed that the variations of first and third frequencies are closest to the experimental values; their use gives the best possible results (solid line in Figure 6): the absolute minimum corresponds to the 5th element and the stiffness reduction is about 27 per cent. It is worth noting that the use of a quantity that is more sensitive to the located damage, such as the 2nd frequency, is not a good choice, as is confirmed by the results obtained with the first two frequencies. This is obvious because the 2nd measured frequency variation is affected by large errors, as revealed by the analytical investigation.

 

Table III. Frequencies and coefficients of variation for the first four modes

| State | $f_1$ (Hz) | $\rho$ (%) | $f_2$ (Hz) | $\rho$ (%) | $f_3$ (Hz) | $\rho$ (%) | $f_4$ (Hz) | $\rho$ (%) |
|---|---|---|---|---|---|---|---|---|
| Undamaged | 71·58 | 0·31 | 279·6 | 0·23 | 620·9 | 0·13 | 1095 | 1·20 |
| Dam A | 71·43 | 0·63 | 278·1 | 0·22 | 619·4 | 0·13 | 1094 | 0·57 |
| Dam B | 70·97 | 1·05 | 275·5 | 0·35 | 616·8 | 0·21 | 1094 | 0·25 |
| Dam C | 70·22 | 0·84 | 270·9 | 0·12 | 612·1 | 0·00 | 1094 | 1·13 |

In the second situation, the experimental data concerns the lowest frequencies of a simple supported steel beam in different conditions, undamaged and damaged. More details of the experimental results can be found in Reference 15, where three sets of beams are examined, Here only beam $B_2$ is analysed. It has a length $l$ of 360 mm and is 30 mm wide and 4 mm high; the centre of the cut is located at 0.25 $l$ from the support. Three kinds of damage are considered: the depth of the notch was increased by a reduction of 12·5 (damage A), 25·0 (damage B), 37·5 per cent (damage C) in the section height, while the width of the cut is taken to be constant and equal to the height of the cross-section.

To inflict damage, the beam must be removed from the supports; notwithstanding the great care taken in constructing the supports, in view of the need to reduce the effects of removal, the frequencies are measured ten times, removing and reinstalling the beam. In Table III the mean values of the first four frequencies and their coefficients of variation $\rho$ are given. In most cases, the values of $\rho_i$ do not reach 1 per cent; they are however generally greater than the frequency changes due to damage. This justifies the expedience of assuming, as observed quantities, the mean values $\bar{f}_i$ of the frequencies over ten tests. The values of $\rho_i$ are used to evaluate the diagonal coefficients $\sigma_i^2 = \rho_i^2 \bar{f}_i^2$ of $\Sigma_n$ while the off diagonal terms are set to zero and it is also assumed that $\Sigma_x$ is proportional to $\Sigma_n$.

A continuous model is adopted and the objective function (12) is evaluated for different items of data and reported in Figure 7. The abscissa $s$ is here measured from the centre of the beam and normalized to $l/2$, so that $s = 1$ indicates a support and $s = 0$ the middle of the beam; the degree of damage is measured by the parameter $x$ as defined in Reference 15. Figures 7(a) and 7(b) refer to damage C and two pairs of frequencies. For the first and second frequencies $\hat{l}(s)$ has only one minimum, apart from one that is not significant at the boundary, and, therefore, gives a unique solution ($s = 0{\cdot}46$, $x = 29$); the first and third frequencies, as expected, give two solutions, one of which is near the centre of the beam and the other, ($s = 0{\cdot}48$, $x = 27$), is very close to the expected solution $s = 0{\cdot}5$ and $x = 29$. Figure 7(c) refers to the objective function $\hat{l}(s)$ evaluated using all the three frequencies; its minimum gives a very satisfactory solution, $s = 0{\cdot}49$ and $x = 28$. An analogous analysis is developed for damage B; using again the first three frequencies, the minimum of the $\hat{l}(s)$, drawn in Figure 7(d), furnishes location and magnitude of damage ($s = 0{\cdot}485$, $x = 63$) very close to the expected values $s = 0{\cdot}5$ and $x = 65$; even in this case of less severe damage, the identified parameters are very good.

## 5. CONCLUSIONS

The present paper addresses the problem of damage detection. It is assumed that damage reduces stiffness in elements or parts of elements of the structure. The paper discusses a new way to tackle

 

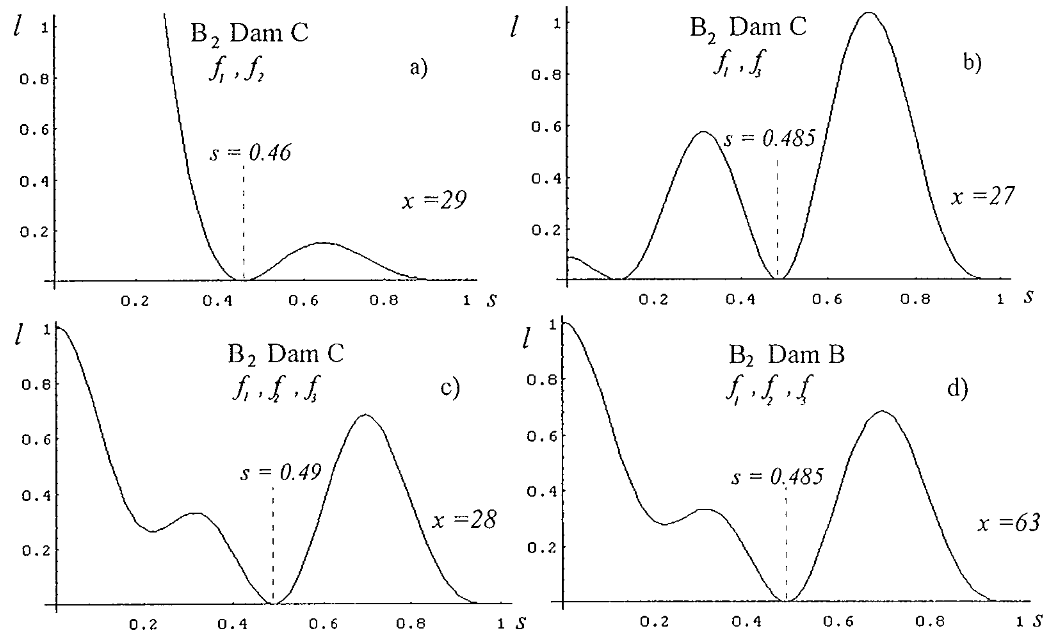


Figure 7. Objective function $\hat{l}(s)$ for the supported beam, for damage B and C

the problem based on the peculiarity that damage affects only a few zones, it is frequently reasonable to suppose that one or at most two elements are damaged. This circumstance justifies separating the problem of damage detection from that of reconstruction in which the whole stiffness distribution must be determined. In this study the damaged zones are unknown but the stiffness of the structure is known to be equal to the undamaged values, except in the few damaged zones. This involves solving a large number of problems, but with a very small number of unknowns. From this viewpoint the problems of damage identification appear simpler than those of reconstruction.

In accordance with numerous damage evaluation procedures in the literature, only frequencies are assumed as modal characteristics and the identification procedure is based on the minimization of an objective function that accounts for the difference between analytical and experimental quantities. The use of frequencies alone cannot be sufficient for structures with relevant symmetries; the problem, is in principle ill posed even for the simply supported beam, where the damaged half of the beam must be known a priori.

Theoretical analysis and numerical investigations indicate that to evaluate the damage of $r$ elements of a discrete structure, $r+1$ measurements are sufficient in a noise-free situation. For the localization of $r$ cracks in a continuous structure, such as a vibrating beam, at least $2r$ frequencies are necessary. In any case, because only a few cracks at a time are usually detected during monitoring, only few measured frequencies are needed. The presence of modelling and experimental errors, however, considerably complicate the problem to reach the exact solution and more frequencies are required to reduce the effect of errors. The key finding remains that the interpretative model has as unknowns, the stiffness of only $r$ candidates among $n$ total members.

 

The procedures developed are applied to sample cases: a supported beam, a clamped beam, a continuous beam and a shear-type frame, using experimental and simulated data. The identification procedure is carried out mainly by the computer code IDEFEM, which includes a finite element general purpose code as a routine and performs all the steps needed to reach the minimum of the objective function.

When experimental data are used, the location of damage is always accurate, though some discrepancies may remain in its quantification, depending on which frequencies are selected from those available. This highlights the sensitivity of the problem to errors and the importance of using reliable experimental data.

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